Equity Yield Curves, Time Segmentation, and Portfolio Optimization Strategies

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(Journal of Financial Planning, forthcoming Nov. 2016)

Executive Summary

- Yield curves exist for equities just as they do for bonds. But equity yield curves become obvious only when *minimum* returns, rather than average returns, are plotted over successively longer time horizons. They establish the importance of time segmentation for equity returns as well as bonds: time is a factor in returns every bit as much as style and size. Optimal allocations thus depend on how time is segmented for planning purposes.

- We utilized CRSP data back to 1928 for returns on U.S. equity asset classes based on style (value, blend, and growth) and size (large, mid, and small cap), plus commodities and real estate. We compare performance of three passive investment strategies optimized with mathematical programming over 1- to 40-year holding periods (120 portfolios overall):

  Strategy A: minimize the worst case scenario (based on Von Neumann's minimax principle in Game Theory – adjusts for risk by focusing on minimum return);
  Strategy B: maximize the Sharpe Ratio (based on Modern Portfolio Theory – adjusts for risk by focusing on volatility);
  Strategy C: maximize expected return (based on classic "expected monetary value" in decision theory – no adjustment for risk).

- Conclusions: 1) All three strategies produce equity yield curves. 2) Strategy A leads to positive minimum returns within 6 years while B and C require 15 years. 3) Strategy C leads to highest average returns but the worst downside. 4) Strategy B falls in between the other two strategies: average returns higher than Strategy A, lower than C, but worse minimum returns than A, better than C.

Introduction

Most financial planners are familiar with bond yield curves. For each maturity date, a bond's yield to maturity (YTM), by definition, is an approximation that is considered the best indicator of the worst case return if the bond is held to maturity.[1] For bonds, these worst case returns are nearly always positive and upwardly sloping. As the time horizon increases, typically so does YTM, though its rise starts to taper off the longer the horizon. The U.S. Treasury publishes them online daily (U.S. Treasury, 2016), and animated videos are available that animate historical bond yield curves (Animations, 2016).

Equity yield curves also exist. That is, the return on equities is also based on the length of time equities are held. But it is not the average return where time matters most.[2] Rather it is the *minimum* return for a given time horizon ("time horizon" and "holding period" are used interchangeably here). These minimum return curves are not exactly the same as bond yield curves, of course. They are not based on fixed cash flows of coupons and redemptions. They simply track the worst case scenario in history for various asset classes over holding periods ranging from 1 to 40 years. But when plotted against time, they closely resemble bond yield curves.

Time is a critical component in personal financial planning. Most financial plans look ahead to identify and develop strategies to meet financial goals over an investor's lifetime. Lifetimes can be broken up into a sequence of years, each with its own specific spending need. Some spending needs are rather easy to predict, such as budgeted living expenses (food, clothing, and shelter), while others are not (emergencies, weddings, layoffs, etc.). The point is that good planning typically requires segmenting life on a year-by-year basis and taking appropriate actions to fund each year.

At a very granular level, bonds themselves present the case for time segmentation. For example, according to dedicated portfolio theory (Leibowitz, 1986; Sharpe, Alexander, Baily, 1995), a retired investor who, through the financial planning process, has identified a target spending need in 7 years should fund that spending need with a bond that matures in 7 years and delivers the precise amount of projected cash need. The 8th year spending need should be funded with the 8-year bond and so on. Institutions, like pension funds, call this "liability driven investing." Liability driven investing seeks to align the timing of a spending need with an asset that can predictably deliver the necessary cash flows when needed – no more, no less. A series of bonds with maturities ranging from 1 to 10 years can therefore fund 10 years of retirement spending with certainty. Each year can be segmented into its own silo where a bond is dedicated to funding that year's need.

Where do equities fit into this picture? Consider a retired investor with a 30 years of retirement income to fund. That means 30 time segments exist. Unfortunately, given the current low interest rates, most retired investors cannot fund 30 years of retirement spending with 30 years of bonds (government, corporate, muni or otherwise). Investors who have a withdrawal rate above the yield curve, which is the case for most retirees, cannot purchase an all-bond portfolio that will last the entire horizon to meet the target spending needs. The cash flows will simply not be enough and the portfolio would fail. Therefore, investors must take on equity exposure to improve the probability of achieving a portfolio sufficient to cover lifetime financial goals. As with bonds, it makes sense that the equity portion of the investor's portfolio should also align with the investor's spending needs, albeit the longer dated spending needs.

In this paper, we examine three strategies to determine which would be more consistent with a time segmentation approach and better support the goals of a financial plan and perhaps work better from a behavioral perspective as well. We tested these strategies over 40 different holding periods, 1-year, 2-year, …, 40-year, to estimate the effect of the length of the holding period on performance

Time Segmentation in Equity Investing

Time segmentation for equities has not been broadly studied. This lack of research is partly due to a crucial difference between institutional investors and individual investors. As Harry Markowitz famously noted in his Nobel Laureate lecture, modern portfolio theory (MPT) was developed primarily for "large (usually institutional) investors." But most individuals are not like institutions. Clearly, they have less money, but they also have finite lifetimes. This makes time horizons inherent to the personal financial planning process and building equity portfolios that best match an individual investor's time horizon becomes critical (see Blanchett, Finke, and Pfau, 2014).

From a practical standpoint, part of the lack of attention regarding equity yield curves may be due simply to the fact that equity yield curves have seldom, if ever, been seen. Unlike the ubiquitous bond yield curve, they simply do not show up in the media – academic journals, trade publications, or anywhere else. Without visuals, they remain an abstract concept seldom considered in any direct way. Equity yield curves, and the importance to time segmentation that they imply, only become obvious when one plots *minimum* returns – the worst case scenarios - that a portfolio might earn over successively longer time horizons.

In the context of a financial plan, periods of poor equity returns put pressure on an investor's portfolio. Markets that produce returns above the target returns for the plan, or even average returns, usually work out fine for the investor. It is bad markets that can cause trouble, especially if they happen early in retirement.

Plotting minimum returns means plotting the worst case scenario over all 1-year periods, 2-year periods, … 40-year periods. Minimum returns may actually be a better way to express the risks that worry investors most - what would happen in the nastiest situation. Figure 1 shows the minimum annualized total returns for the S&P 500 over holding periods of 1, 5, 10, 20, 30 and 40 years, based on a historical audit from 1928 to 2015 using data from the Center for Research in Security Prices (CRSP).

Figure 1

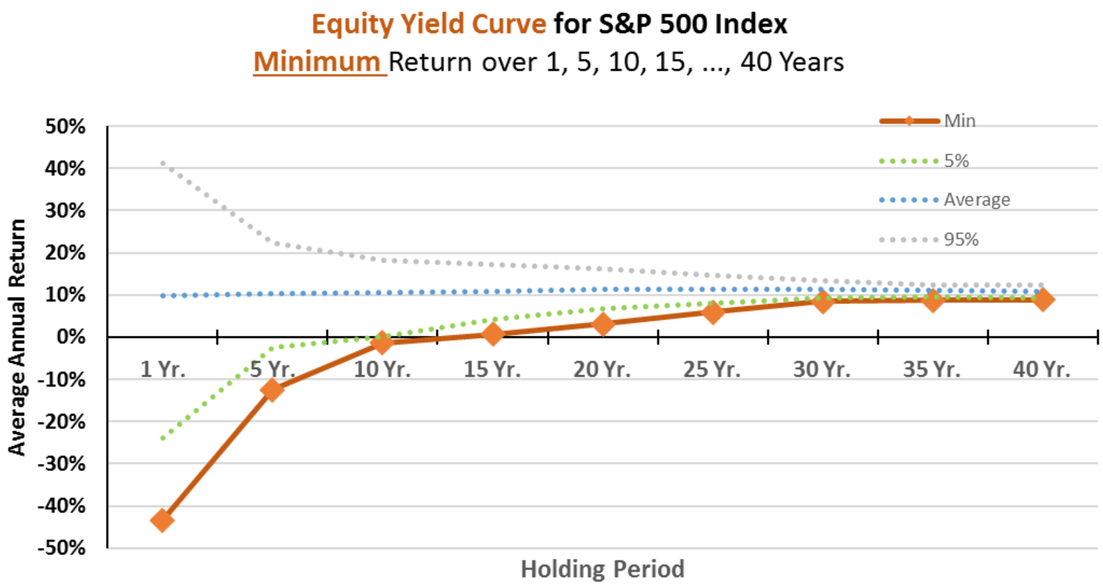


Figure 1 is based on Table 1, which shows average and minimum returns and when they occurred. Not surprisingly, the worst 10-year span was 1999-2008 when the index lost 1.4 percent per year. Not shown is the second worst 10-year span, losing 0.95 percent per year from 2000 to 2009, nor the third worst losing .89 percent per year from 1929 to 1938. Also, it is not surprising that the Great Depression was responsible for all of the other worst case scenarios.

S&P 500 Index - Annualized Returns Over All Spans of Given Length, 1928-2015

	Average Returns		Minimum Returns		
Length of Span in Years	Average Annualized Growth Rate	Ending Value Over Period of $100,000	Minimum Annual Growth Rate	Ending Value Over Period of $100,000	Year(s) Minimum Happened
1	9.7%	$109,721	-43.3%	$56,651	1931
5	10.2%	$162,727	-12.5%	$51,374	1928-1932
10	10.5%	$270,795	-1.4%	$87,008	1999-2008
15	10.9%	$469,004	0.6%	$110,107	1929-1943
20	11.2%	$843,018	3.1%	$184,361	1929-1948
25	11.4%	$1,471,234	5.9%	$419,325	1929-1953
30	11.3%	$2,464,348	8.5%	$1,147,279	1929-1958
35	11.1%	$3,976,467	8.7%	$1,835,156	1929-1963
40	10.9%	$6,307,184	8.9%	$2,976,168	1929-1968

Table 1

Three Strategies Based on Investment Goals

What strategy will achieve an optimal portfolio? Unfortunately, a unifying, all-encompassing strategy does not – cannot – exist because the "optimal" strategy depends on the goal for the portfolio. Tradeoffs, like the familiar volatility/return tradeoff, must be made.

The three strategies listed below are all designed to optimize different goals for a portfolio:

Strategy A - minimize the worst case scenario
Strategy B - maximize the Sharpe ratio
Strategy C - maximize expected returns

Strategy A: Minimize the worst case scenario (The "Minimax" Principle)

Strategy A is the most conservative from a financial planning perspective. It considers "risk" to be the worst return that the portfolio has ever earned for a given horizon, not the volatility of its returns. Strategy A seeks the allocation that minimizes the worst case returns for each time horizon. If the portfolio return were actually negative, it would seek to make the loss as small as possible. In other words, Strategy A seeks to minimize the maximum loss (worst case). Hence the name "minimax."

Planners familiar with Bengen's "SAFEMAX" concept are well aware of this idea. SAFEMAX, otherwise known as "the 4 percent rule," indicates the highest initial withdrawal (plus the prior year's inflation) that a retiree can pull from a 50-50 stock/bond portfolio that will be sustainable over a 30-year span. That is, even in the worst 30-year period, the portfolio would not have failed until after 30 years had passed.

The minimax principle is rooted in the work of John von Neumann, whom many believe was the greatest mathematician of the 20th century (Halmos, 1980; MacRae, 1999). He was one of the founding fathers of the field of decision analysis and game theory. A contemporary of Albert Einstein at Princeton, he is credited with discovering the minimax principle. The idea in decision theory is that, when deciding among several alternative courses of action, some of which could result in a loss, it is optimal to make the best of a bad situation by choosing the action that will lose the least if the worst case scenario does, in fact, happen. Abraham Wald also contributed to this theory (Wald, 1944). Since it is poor market returns that put stress on a financial plan, the minimax solution in portfolio theory is quite congruent with the goal of most conservative investors.

The differences between the average and worst performance can be dramatic. For example, Table 1 reveals that over the average 10-year period, the S&P 500 gained 10.5 percent per year. $100,000 would have grown to $270,795. But over its worst span, 1999-2008, it lost 1.4 percent per year ending at $87,008. After inflation, it would be even lower in real terms. The differences between average and minimum returns are larger for midcap (12.0 percent and -2.1 percent respectively) and small cap investments (12.9 percent and -2.6 percent). It does not take much imagination to see why investors expecting "average" results would be very concerned if they happen to invest during the worst bad period or even a poor one.

This intuitive insight is backed up by the behavioral economics concept known as Prospect Theory (Kahneman and Tversky, 1979). According to this theory, most investors, especially conservative investors, dislike losses more than they favor gains, and would likely be quite concerned about the worst case scenario. It is therefore reasonable that they choose an investment strategy that seeks to minimize the damage that a worst loss could create. Among the first to apply the minimax principle to investing was Martin Young, who examined it from the standpoint of management science and utility theory (Young, 1998).

Strategy B: Maximize Sharpe Ratio
The second strategy seeks to maximize the “efficiency” of the portfolio according to Modern Portfolio Theory, utilizing a mean-variance optimization routine that seeks to maximize the Sharpe Ratio. Whether suitable for personal financial planning or not, it remains the most common strategy used in the industry. The Sharpe Ratio subtracts the return of a risk-free asset from the actual return each year (or quarter or month) of some asset and divides the result by the standard deviation, all over the same time period. To a statistician, the Sharpe Ratio is the inverse of the “coefficient of variation” after making adjustments for the risk-free asset.

Sharpe Ratio = $\frac{r_p - r_f}{\sigma_p}$

Coefficient of Variation = $\frac{\sigma_p}{r_p}$

r_p = Average Portfolio Return

r_f= Risk Free Return (T-bills)

σ_p= Standard Deviation of Portfolio Returns

A portfolio whose allocations among the available asset classes have the highest Sharpe Ratio is considered to be an “efficient” portfolio because it is delivering the highest possible expected return per unit of volatility. The risk-free return is typically the 30-day Treasury bill rate, though some argue it should be the rate on one-year TIPS.[3] The idea is to maximize the portfolio return per unit of volatility to obtain the best “risk-adjusted” return. It should be noted that Strategy A (minimax) could also be considered a “risk-adjusted” return in the sense that it factors in the worst case returns, not just the variability of returns.

Strategy C: Maximizing Expected Return
Strategy C has the simple goal of maximizing average annual return, ignoring risk. This approach is often taken by advisors who ignore time segmentation issues, claiming that by generating sufficient returns investors will be able to reach their goals without concern for risk measures. Markowitz noted that a portfolio seeking to maximize expected return would simply allocate to the single highest return asset (Markowitz 1991). This was indeed the case in our empirical investigation of Strategy C, which allocated only to small cap value stocks, the highest returning asset class in the data set, over all 40 holding periods.

Comparing the Strategies for 1 to 40 year Horizons
How well do these strategies compare to each other and compared to the benchmark of 100% S&P 500 index?

To answer this question, a large number of portfolios were constructed using Excel’s “Generalized Reduced Gradient” nonlinear programming algorithm. Each strategy was optimized over each time horizon (120 optimizations overall), and three performance measures were tracked: 1) average (mean) return, 2) minimum return, and 3) standard deviation of the ending value.[4]

Each portfolio was rebalanced annually. The analysis was a historical audit to maintain the sequence of returns that actually occurred from 1928 to 2015, including the Great Depression. Data for 16 different US asset classes from the Center for Research in Security Prices (CRSP) and the Fama/French data set (French 2016) were used: S&P 500, CRSP 1-2 Growth, CRSP 1-2 Neutral, CRSP 1-2 Value, CRSP 3-5 All Midcap, CRSP 3-5 Midcap Growth, CRSP 3-5 Midcap Neutral, CRSP 3-5 Midcap Value, CRSP 6-8 All Small Cap, CRSP 6-8 Small Cap

Growth, CRSP 6-8 Small Cap Neutral, CRSP 6-8 Small Cap Value, CRSP 9-10 All Microcap, CRSP 1-10 Total Market, Commodities (based on CRB Index), Real Estate (based on FTSE NAREIT Composite Index).

International and emerging markets were excluded based on the assumption that some might question the accuracy for returns of these two asset classes for a study going back as far as 1928. This would be especially true for emerging markets, which clearly are different now than they were then. A limitation on these results, therefore, is that the conclusions apply only to the portion of a portfolio allocated to US equities.

Another limitation is that the goal was to find the best portfolio for each holding period based on returns over the entire time span from 1928 to 2015. It could be that the portfolio allocations would change and produce different results if returns for 1947-2015 or some other starting date were analyzed. Such extensions await future research.

Results: The Equity Yield Curves

As mentioned earlier, a separate non-linear programming formulation was solved for each strategy and each time period of 1 year to 40 years, 120 scenarios in total. All returns are total returns, compounded annually. For time segments longer than one year, the returns are computed as compounded annualized averages within the span, but the average for all, say, 10-years spans is the simple mean because compounding can only be done within each 10-year span.

Table 2 is the key table in this analysis. It presents performance results for each strategy and the S&P 500 Index as benchmark over 5, 10, 15, 20, 25, 30, 35, and 40-year rolling time spans. Equity yield curves are quite evident for each strategy in Figure 2, which plots both averages and minimums from Table 2 including all 40 time spans.

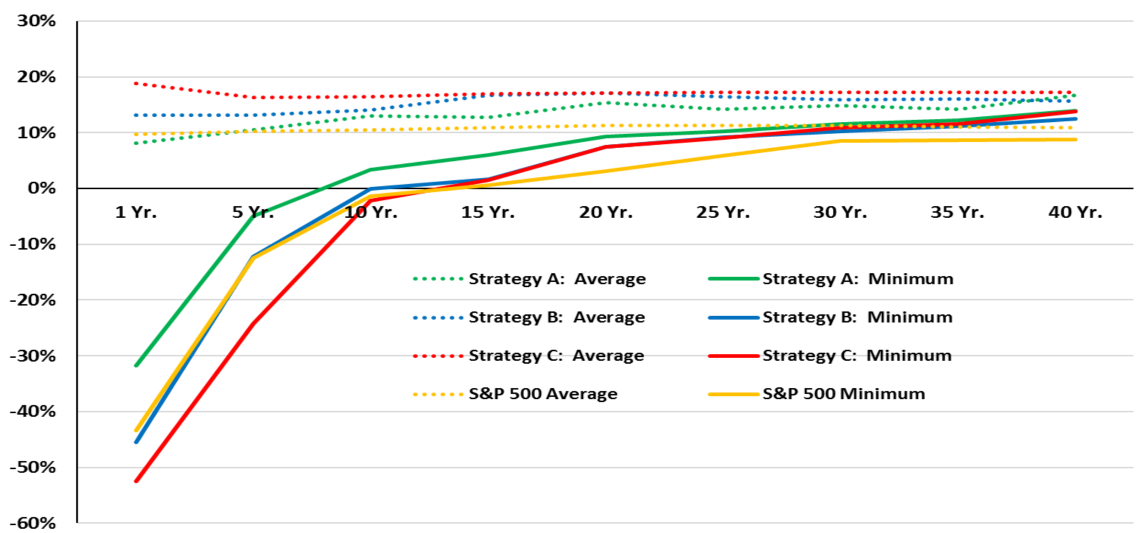
All results are as expected in all cases from 1 to 30 years. As is visibly obvious, Strategy A (Minimax) achieves the best minimums but at the expense of lower averages. Strategy C (Expected Return) achieves the best averages and worst minimums, and Strategy B (Maximize Sharpe Ratio) falls in between. This is true also for the standard deviations. For 40-year time spans, Strategy B (Sharpe) does a little worse than Strategy A in terms of average return but its standard deviation is a tie. Comfortingly, all three strategies beat the S&P 500 benchmark in all but a few cases.

Some might wonder why the S&P 500 does not do better. First, It is difficult to second guess these results because they examine something new – holding periods longer than one year. All (or most) prior research has been based on one-year holding periods, and it is this prior research has formed most planners' intuitions. Building optimal portfolios (based on mathematical programming, a well-known optimizing algorithm) for time spans longer than one years is sailing in uncharted waters and could well produce results that may seem counterintuitive. There is also some research that questions whether or not a pure S&P 500 Index should be considered an efficient portfolio (Grinold, 1992).

Table 2
Performance Statistics over Selected Time Spans

Performance Statistic	Strategy	Average (Mean) of All 40 Spans	5 Yr.	10 Yr.	15 Yr.	20 Yr.	25 Yr.	30 Yr.	35 Yr.	40 Yr.
Average (Mean) Returns	Strategy A	13.80%	10.5%	13.1%	12.8%	15.4%	14.2%	14.8%	14.2%	16.7%
	Strategy B	15.50%	13.2%	14.1%	16.8%	17.1%	16.5%	15.9%	16.1%	15.6%
	Strategy C	17.00%	16.3%	16.5%	17.0%	17.2%	17.2%	17.2%	17.3%	17.2%
	S&P 500	10.90%	10.2%	10.5%	10.9%	11.2%	11.4%	11.3%	11.1%	10.9%
Minimum Returns	Strategy A	5.60%	-4.9%	3.4%	6.0%	9.3%	10.3%	11.5%	12.3%	13.9%
	Strategy B	2.30%	-12.2%	-0.1%	1.6%	7.5%	9.2%	10.3%	11.2%	12.5%
	Strategy C	0.50%	-24.2%	-2.2%	1.5%	7.5%	9.1%	11.0%	11.6%	13.9%
	S&P 500	0.00%	-12.5%	-1.4%	0.6%	3.1%	5.9%	8.5%	8.7%	8.9%
Std. Deviation of Returns	Strategy A	0.04	0.08	0.05	0.04	0.03	0.02	0.02	0.01	0.01
	Strategy B	0.05	0.09	0.05	0.04	0.03	0.03	0.02	0.01	0.01
	Strategy C	0.06	0.13	0.06	0.04	0.03	0.03	0.02	0.02	0.02
	S&P 500	0.04	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0.01

Figure 2
Averages and Minimums for Optimal Portfolios from Table 2
Strategy A - Minimize Worst Case (Minimax)
Strategy B - Maximize Sharpe Ratio
Strategy C - Maximize Expected Return
S%P 500



It is visibly obvious that the equity yield curve for Strategy A is higher than B and B is higher than C. Standard regression analysis of the minimums against the natural log of the years (Y = a + b(lnX) yields Adjusted R Squared values in the 90-95 percent range and _P-values well below 5 percent for the coefficient. However, we are dealing with rolling horizons with overlapping data points, meaning the data observations are not independent of each other. This violates one of the underlying assumptions in the mathematical mechanics of the econometric theory behind regression analysis, so, technically speaking, the results do not meet the standard needed to claim "statistical significance."

A more complete version of Table 2 (available on request from the authors) reveals that Strategy A (Minimax) reaches a positive minimum return and stays positive for all time spans longer than 6 years. This easily beats the other two strategies and the S&P 500 – they all require 15 years to achieve this milestone. In the context of an investor's financial plan, the Minimax strategy was able to recover more quickly from stock market losses, thereby helping the portfolio to be more resilient in times of market turmoil.

Limitations of Mathematical Optimization

As shown in Table 2, Strategy C (Expected Return) produces average returns that are higher than the other strategies. Why not use it exclusively?

First, its worst case becomes significantly worse than the other two strategies. This disadvantage dwindles as the holding period extends but never quite vanishes. Some investors claim they can tolerate large swings in their portfolios. But when actually faced with large losses, they may not have the fortitude to ride through the market turmoil and sell at just the wrong time. This greater downside risk may also have a large impact on the probability of achieving their long term financial goals since it is low returns from poor markets that put pressure on a financial plan, not average returns.

The second and perhaps more provocative reason against Strategy C is the fact that its mandated allocation does not follow conventional wisdom regarding diversification. As Markowitz predicted, it consists of 100% small cap value stocks over all horizons. It may be mathematically correct, but few investors would feel comfortable with a portfolio constructed entirely of small cap value stocks. This lack of diversification among asset classes is a common side effect of mathematical programming. Unless constrained, optimizers blindly pursue mathematical solutions - they ignore diversification, conventional wisdom, possible end-client reactions, and other "human" factors.

Regarding allocations mandated by the mathematics for Strategies A and B, there are no easiliy discernable patterns – each holding period generates a different allocation for each strategy. Only Strategy C led to a single asset class allocation. The number of asset classes used changed as well as the allocations to each asset class varied from one holding period to the next. This underlines the importance of time segmentation in building portfolios.

Strategy A (minimax) demonstrated the greatest degree of diversification in that some of the 40 different portfolios used all 16 asset classes (13, 14 and 17 year holding periods) while others used only two of the asset classes (5, 20, 23, 24, 29, 37, and 39 years). On average, Strategy A used 8.4 asset classes. A plot of the number of asset classes against holding period lengh follows what could be best described as a random walk following no distinct path. The asset class that had the largest average allocation over all 40 horizons for Strategy A was Large Cap

Value (33.0%), followed by Small Cap Value (28.4%) and Microcap (10.8%). The balance of the allocations were scattered among all the other asset classes. The asset classes that averaged close to zero allocations were the S&P 500, CRSP 1-2 Neutral, Midcap Growth, Total Market, and Real Estate.

Strategy B (Sharpe) was a little better behaved. It generated diversified portfolios for horizons shorter than 15 years but concentrated on only two for most of the periods longer than 15 years. It used a maximum of six asset classes for one holding period (5 years), five asset classes for eight holding periods (3 years and 6-12 years), four asset classes for three holding periods (4, 13, and 14 years), and three asset classes for five holding periods (1, 2, 18, 19, and 40 years). All other holding periods had only two asset classes, though not the same two. As expected, small cap value and midcap value were the dominant pair in all horizons, showing up in nearly every holding period. They averaged 43% and 40% respectively over all 40 holding periods combined. Small cap neutral (9%) and Large cap growth (7%) were in distant third and fourth places.

Future research with shorter time periods that 1928-2015 may reveal more discernable allocation patterns for Strategies A and B.

The Tradeoff: Average vs. Minimum Return

The minimax Strategy A should be attractive to conservative investors because it provides the highest possible minimum returns over a given time horizon. It clarifies the consequences for investors who harbor fears that a bad market will create havoc in their lives. Basing all other calculations on a foundation of worse case returns means any performance surprises should be pleasant – at least better returns than other strategies. There is no guarantee that the future can never be worse than the past, of course, but an advisor facing an angry client can always point out that the investment strategy was based on the most prudent actions possible, at least from a historical perspective.

The downside, of course, is that this prudence comes at a price - lower *average* returns. In a sense, the minimax strategy is like any other insurance policy – it is not free (the strategy is long only in that it uses neither hedging nor portfolio insurance strategies). Table 2 reveals the tradeoff between higher averages versus higher minimums over all horizons. For example, the expected return for a 10-year optimal portfolio based on Strategy A is 13.1 percent but it could be as low as 3.4 percent. For Strategy B, the equivalent returns are 14.1 percent and -.1 percent respectively. In other words, giving up 1.0 percent in average return increases minimum return by 3.5 percent in the 10-year case. Another way to say this is that to gain a 1 percent increase in the minimum, the investor would have to give up about 29 basis points (“bps”) in average return (1/3.5 = .286).

Unfortunately, this tradeoff is not constant. It is fairly well behaved for horizons of 15 years or shorter. The tradeoff rises slowly but steadily as the time horizon lengthens from a low of 13 bps for all 2-year spans to a high of 91 bps for all 15 year spans, with an average of about 47 bps. Beyond 15 years, the pattern becomes quite random, averaging 122 bps.[5]

As expected, an advisor choosing between Strategies A or B for a client’s lifetime plan would need to know the time horizon relevant to each client as well as his or her risk tolerance. Some clients may feel that they would rather have the assurance that their equity portfolio will not cause them any problems because their entire equity portfolio has been built to withstand the worst case scenario for their own horizon. Other clients may be willing to separate their portfolio into segments by time. They would use the minimax strategy for horizons of 15 years or less, and use the maximum average return strategy for 16 years or longer (assuming they can tolerate all small cap stocks).

The Fixed Income Component

The foregoing analysis ignored fixed income investments. It focused only on the equity component of a client's portfolio. The equity allocation could be called the client's "Growth Portfolio" because its purpose is grow over time. Some investors in the accumulation stage may be quite happy to stop there. But for other investors who worry about losses or those approaching retirement, the role of fixed income becomes increasingly important.

The fixed income component in a portfolio is generally designed for a different purpose: stability. For retirees, it can actually serve two purposes simultaneously. Bonds can provide both stability and predictable income using what is known as dedicated portfolio theory. It consists of buying and holding individual bonds to maturity in just the right quantities and maturities so that coupon interest and redemptions combine together each year to match the income needed. Financial textbooks call this the "cash matching problem." The trick for this "Income Portfolio" is figuring out the quantity of each maturity to buy so as to get the correct match. Fortunately, mathematical algorithms to accomplish this have already been developed so that the bond cash flows not only match the income stream, they do so by using the least costly set of bond portfolio needed to accomplish the task (Huxley and Burns, 2005).

Conclusion and Further Research

Time segmentation can and should play an important role in developing a portfolio that aligns with an investor's financial plan. When applied to equities, a new yield curve becomes apparent. A growth oriented equity investment strategy to minimize the worst case scenario rather than average or expected returns takes advantage of this phenomenon and represents an attractive option for conservative investors. When integrated with dedicated portfolio theory on the fixed income side, the combination of the two approaches should represent a welcome response to the fear expressed most often by those near or in retirement: running out of money.

Future research should explore the nuances of dedicated portfolio theory, integrating equity portfolio strategies, sustainable withdrawal rates, , and the critical path a retirement portfolio must follow to maximize the probability of lasting a lifetime. It would also be interesting to explore the robustness of these conclusions by testing them using shorter historical data bases. Would the results be the same if replicated back to 1947 or some other starting date? Extensions of this research should reveal the answers to these types of questions.

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[1] Technically, the YTM calculation assumes that all coupon proceeds are instantly reinvested in zero coupon bond purchased at precisely the same rate as the original YTM.

[2] There is a statistically significant effect of holding period on average returns, but its effect is negligible compared to its effect on minimum returns.

[3] The correlations between T-bills and TIPS are weak for short holding periods. But as the length of the holding period extends, the correlation tends to strengthen. For all 40 holding periods of 1 to 40 years, the average correlation is .65 with a high of .91 (17-year holding period) and a low of .15 (1-year holding period). These correlations suggest that the results for Strategy B would not likely change significantly from those shown here, though the question suggests further research in the future.

[4] Maximum and median returns were also tracked but are not shown. Maximums are rather irrelevant and were tabulated primarily as a matter of personal curiosity. Median returns had a 90% or higher correlation with mean returns and were thus considered redundant with the mean returns shown here.

[5] Strategy A (Minimax) dominates Strategy B (Sharpe) – where both average and minimum returns are better - for holding periods of 28 years and for holding periods of 37 to 40 years.